Journal of Engineering Physics and Thermophysics, Vol. 70, No. 6, 1997

CALCULATIONS OF TURBULENT JETS WITH FINELY DISPERSED SOLID ADMIXTURES

E. F. Avdeev and I. A. Chusov

UDC 532.529

We suggest a computational-experimental method for determining the hydrodynamic characteristics of a dust-laden gas jet. The method is based on Prandtl–Mieses generalized variables. For this case we obtain an algebraic model of turbulence which allows us to take into account the mutual effect of phases and the initial conditions at the nozzle outlet.

Methods of calculation for a jet with a finely dispersed solid admixture are in the development stage at present [1]. Experimental data [2, 3] show that the presence of an admixture substantially changes the dynamic pattern of flow and is evidenced by an increase in the range of the jet and in its contraction. This indicates that an admixture influences both the turbulent jet structure and the character of the processes of turbulent mixing [4].

Analysis of the literature shows that the models of turbulence for free gas disperse flows can be divided into two groups. The first group involves generalization of the Prandtl mixing path length model. Here we should distinguish a model which makes it possible to obtain expressions for both the coefficients of turbulent viscosity and the turbulent analog of the Schmidt number proceeding from the same assumptions [1]. The sole inconvenience of the model described in [1] is the dependence of ε and ε_s on the magnitude of the eddy viscosity of a pure gas ε_0. This fact necessitates the inclusion in calculation of the problem for a pure gas jet with boundary conditions similar to those of the basic problem. For example, in [5] the problem of an axisymmetric jet was additionally solved and the turbulent viscosity coefficient ε_0 was determined from Sekundov's one-parameter model [1]. The second group of models is characterized by the absence of a general approach to the selection of hypotheses for the coefficients of turbulent transfer of phases. The correlation terms in the conservation equations for the carrying gas are simulated by two-parameter models, whereas for correlations of a "gas" of particles either additional relations are introduced or they are assumed to be proportional to the correlations for the gas [6]. Calculations carried out by employing the models of the first and second groups show a satisfactory agreement with the available experimental data. Nevertheless, the models of the first group are the most popular, which is probably due to the simplicity of the closing expressions and to the presence of a clear physical interpretation.

At the present time, for simulating the behavior of heterogeneous mixtures with solid or liquid droplet admixtures the model of interpenetrating continua [7] has gained the widest use. According to this model, a discrete phase is replaced by a continuous medium with its own set of continuum parameters: density, velocity, etc. Moreover, to simulate the dynamic characteristics of a "gas" of particles, its own momentum and continuity equations are written. To obtain a numerical solution of the equations of transfer for a solid admixture, the methods can be used which are applied for solving ordinary boundary-layer equations.

Calculation of the parameters of a polydisperse flow, when the admixture is represented by a large number of particle fractions, generates a need for solving the corresponding conservation equations, which leads to a considerable increase in computation time. One of the possible means of decreasing computational expenditures is application of the generalized analog of the Prandtl–Mieses transformation to conservation equations. This transformation is rather often and successfully applied to calculations of flows in a boundary-layer approximation, but it was used for the first time to calculate dust-laden gas jets. Among the advantages of this approach there are: 1) elimination of the transverse velocity component; 2) possibility for transforming initial equations to the form of heat conduction equations due to which the singularity at the point $x = 0$ is eliminated.

Obninsk Institute of Atomic Power Engineering, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 70, No. 6, pp. 919-923, November-December, 1997. Original article submitted August 14, 1995.

1062-0125/97/7006-0878$18.00

We shall consider an axisymmetric turbulent discharge of a gas with solid particles from a nozzle of radius R_0 into a flooded space filled with a quiscent gas with the same physical properties. In this case the following assumptions are made: a) it is assumed that the dust-gas mixture is continuous and is described by a model of a two-velocity solid medium; b) slippage is taken into consideration only in the longitudinal direction; c) interaction between solid particles and the gas is taken into account by the resistance force; d) the admixture is represented by spherical particles of the same diameter.

The equations of an axisymmetric stationary flow for the carrying gas and for the "gas" of particles in a boundary-layer approximation have the form:

$$\frac{\partial}{\partial x}(r\rho U) + \frac{\partial}{\partial r}(r\rho V) = 0\,, \tag{1}$$

$$\rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial U}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\rho\varepsilon \frac{\partial U}{\partial r}\right) - NF_x\,, \tag{2}$$

$$\frac{\partial}{\partial x}(r\rho_s U_s) + \frac{\partial}{\partial r}(r\rho_s V_s^*) = 0\,, \tag{3}$$

$$\rho_s U_s \frac{\partial U_s}{\partial x} + \rho_s V_s^* \frac{\partial U_s}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\rho_s\varepsilon_s \frac{\partial U_s}{\partial r}\right) + NF_x\,, \tag{4}$$

$$\rho_s U_s \frac{\partial \rho_s}{\partial x} + \rho_s V_s^* \frac{\partial \rho_s}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\rho_s\varepsilon_s}{\mathrm{Sc_t}} \frac{\partial \rho_s}{\partial r}\right)\,, \tag{5}$$

where

$$V_s^* = V_s + \rho_s' V_s'/\rho_s\,.$$

The force of interphase interaction is determined by the expression

$$NF_x = \rho\, C_d \rho_s (U - U_s)\, |U - U_s| / (\rho_p\, d)\,.$$

The drag factor, taking account of the correction for the concentration of the flow, is [7]

$$C_d = (24/\mathrm{Re}_p + 4/\sqrt{\mathrm{Re}_p} + 0.4)\,(1 - \rho_s/\rho_p)^{-2.7}\,.$$

The boundary conditions are written proceeding from asymptotic boundary-layer notions:

$$r = 0 \quad \frac{\partial U}{\partial r} = \frac{\partial U_s}{\partial r} = \frac{\partial \rho_s}{\partial r} = 0\,,$$
$$r \Rightarrow \infty \quad U,\, U_s,\, \rho_s \Rightarrow 0\,. \tag{6}$$

The system of equations (1)-(5) is augmented with integral conservation conditions of the momentum of the mixture and the mass of the admixture:

$$I_0 = 2\pi \int_0^\infty (\rho U^2 + \rho_s U_s^2)\, r dr\,, \quad G_0 = 2\pi \int_0^\infty \rho_s^2 U_s\, r dr\,. \tag{7}$$

The stream functions for both phases are introduced satisfying the continuity equations (1) and (3):

$$rpU = \frac{\partial\psi}{\partial r}, \ rpV = -\frac{\partial\psi}{\partial x}, \ r\rho_s U_s = \frac{\partial\psi_s}{\partial r}, \ r\rho_s V_s^* = -\frac{\partial\psi_s}{\partial x}, \tag{8}$$

and the new longitudinal coordinates are written in the form:

$$d\xi = \int_0^x \rho^2 \varepsilon dx, \ \xi_1 = \int_0^x \rho_s^2 \varepsilon_s dx, \ \xi_2 = \int_0^x \frac{\rho_s^2 \varepsilon_s}{\mathrm{Sc}_t} dx. \tag{9}$$

Applying formulas for changing to new variables:

$$\frac{\partial}{\partial x} = a\frac{\partial}{\partial b} - c\frac{\partial}{\partial d}, \ \frac{\partial}{\partial r} = e\frac{\partial}{\partial d},$$

where

$$a = \begin{vmatrix} \rho^2 \varepsilon \\ \rho_s^2 \varepsilon_s \\ \rho_s^2 \varepsilon_s / \mathrm{Sc}_t \end{vmatrix}, \ b = \begin{vmatrix} \xi \\ \xi_1 \\ \xi_2 \end{vmatrix}, \ c = \begin{vmatrix} r\rho V \\ r\rho_s V_s^* \\ r\rho_s V_s^* \end{vmatrix}, \ d = \begin{vmatrix} \psi \\ \psi_s \\ \psi_s \end{vmatrix}, \ e = \begin{vmatrix} r\rho U \\ r\rho_s U_s \\ r\rho_s U_s \end{vmatrix},$$

the system of equations (1)-(5) is written in the form

$$\frac{\partial U}{\partial \xi} = \frac{\partial}{\partial \psi}\left(I_g U \frac{\partial U}{\partial \psi}\right) - f\rho_s (U - U_s) \, |U - U_s| / (\rho^2 \varepsilon U), \tag{10}$$

$$\frac{\partial U_s}{\partial \xi_1} = \frac{\partial}{\partial \psi_s}\left(I_s U_s \frac{\partial U_s}{\partial \psi_s}\right) - f\rho (U - U_s) \, |U - U_s| / (\rho_s^2 \varepsilon_s U_s), \tag{11}$$

$$\frac{\partial \rho_s}{\partial \xi_2} = \frac{\partial}{\partial \psi_s}\left(I_s U_s \frac{\partial \rho_s}{\partial \psi_s}\right). \tag{12}$$

Here

$$f = \frac{3C_d}{4\rho_p d}, \ I_g = 2\int_0^\infty \frac{d\psi}{\rho U}, \ I_s = 2\int_0^\infty \frac{d\psi_s}{\rho_s U_s}.$$

The boundary conditions in the transformed coordinates are analogous to Eq. (6):

$$\psi = \psi_s = 0 \quad \frac{\partial U}{\partial \psi} = \frac{\partial U_s}{\partial \psi_s} = \frac{\partial \rho_s}{\partial \psi_s} = 0,$$

$$\psi, \psi_s \Rightarrow \infty \quad U, U_s, \rho_s \Rightarrow 0.$$

The solution of system of equations (10)-(12) was found numerically. To obtain the finite-difference analogs of the transformed equations, we used an implicit four-point scheme [8]. The linearization of the discrete analogs was made by the method of retarding coefficients. The global iteration process was constructed on the basis of the method of separate pivots [8].

For numerical solution we selected the coordinates (ξ_2, ψ_s) as the basic ones, since Eq. (12) does not contain turbulent transfer complexes. For this equation to be solved, we constructed a computational grid which is uniform in the longitudinal and lateral directions. The computational domain for solving Eqs. (10) and (11) was constructed on the basis of relations (8) and (9). The practice of calculations showed that application of the Prandtl–Mieses

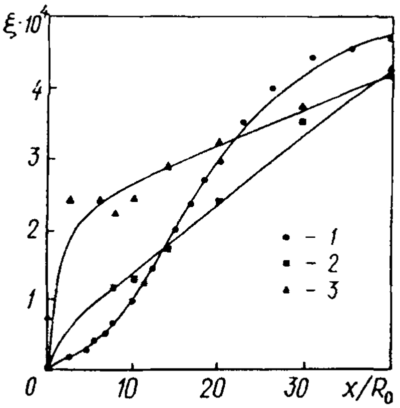

Fig. 1. Computational-experimental dependences of longitudinal coordinates: 1) concentration of a solid admixture, 2) concentration of carrying gas, 3) concentration of "gas" of particles. Curves, their approximations.

generalized transformation made it possible to obtain a 30% reduction in the time needed for one step along the marching coordinate.

Equations (10) and (11) preserve the unknown complexes $\rho^2\varepsilon$ and $\rho_s^2\varepsilon_s$ that characterize the intensity of turbulent transfer, as well as the complex $\rho_s^2\varepsilon_s/\mathrm{Sc_t}$ in implicit form. The expressions for these complexes can be obtained by the following computational-experimental technique. Equations (10)-(12) were solved using, as the initial values, the complexes $\rho^2\varepsilon$, $\rho_s^2\varepsilon_s$, and $\rho_s^2\varepsilon_s/\mathrm{Sc_t}$, their values calculated by means of a model suggested in [6] for the distribution of the parameters at the nozzle outlet [1]. Then, in each step of calculation we selected iteratively values for which the integral conservation conditions (7) were satisfied. Using the results of calculation and the experimental data of [2], we constructed computational-experimental relations that connected the transformed and physical longitudinal coordinates for the three parameters U, U_s, and ρ_s. A correspondence between the transformed and physical coordinates was established by coincidence of the calculated and experimental values of the parameters. The relations constructed in this way were approximated by the series:

$$\xi = a_0 x + \sum_k a_k \arctan(kx)\,, \quad \xi_1 = b_0 x + \sum_k b_k \arctan(kx)\,,$$
$$\xi_2 = c_0 x + c_1 \ln(z + x) + \sum_k \frac{-c_k}{k(z + x)^k}\,. \qquad (13)$$

The values of the coefficients a_k, b_k, c_k, and z were found by the least-squares method. To approximate the graphical functions $\xi(x)$, $\xi_1(x)$ it turned out to be sufficient to take into account two terms of the series and seven for $\xi_2(x)$. Computational-experimental dependences for ρ_s, U, and U_s are presented in Fig. 1. The curves represent their approximations by series (13). In accordance with Eq. (9), simple differentiation of series (13) gives expressions for the unknown complexes:

$$\rho^2\varepsilon = a_0 + \sum_k \frac{ka_k}{(1 + k^2x^2)}\,, \quad \rho_s^2\varepsilon_s = b_0 + \sum_k \frac{kb_k}{(1 + k^2x^2)}\,,$$
$$\rho_s^2\varepsilon_s/\mathrm{Sc_t} = c_0 + \sum_k \frac{c_k}{(z + x)^k}\,. \qquad (14)$$

Expressions (14) acquire physical meaning in the case of passage to the limit $x \Rightarrow 0$ or $x \Rightarrow \infty$. In the first case, the sum of the coefficients of the series will reflect the initial turbulence of the jet. The passage $x \Rightarrow \infty$ corresponds to the process of the degeneration of turbulence, and the values of the coefficients a_0, b_0, and c_0 correspond to the level of turbulence at an infinite distance from the jet exit.

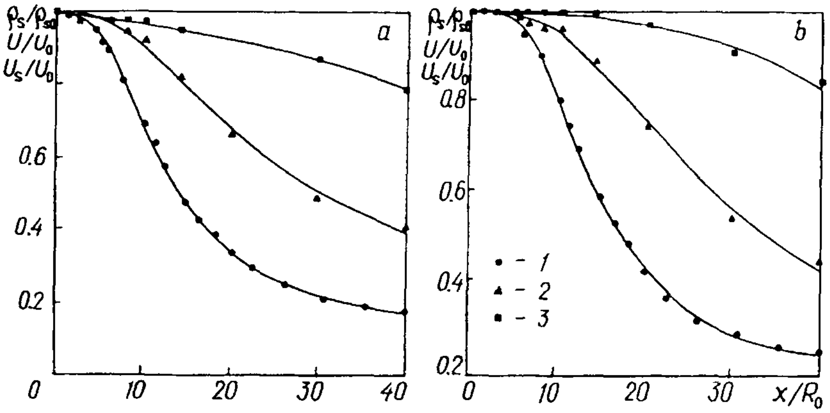

Fig. 2. Comparison of the results of calculation with the experimental data of [2] for particles with sizes of 45 (a) and 67 μm (b): 1) concentration of solid admixture, 2) gas velocity, 3) velocity of "gas" of particles. Points, experiment; curves, calculation.

For checking the applicability of expressions (14) to the simulation of gas-dust jet flows, we carried out calculations for different particle sizes. In this case the dynamic and geometric parameters of the jet at the nozzle outlet corresponded to the conditions of the experiment of [2]. A comparison of the axial distributions of the velocities for both phases and of the concentration of the admixture for particles with size $d = 45$ μm is given in Fig. 2a. The curves show the results of calculation. It is seen that the coincidence of the experimental and calculated values of the parameters is rather good. Figure 2b compares the calculated and experimental dynamic characteristics of a jet along the symmetry axis for particles with diameter $d = 67$ μm. It is seen that an increase in partice size, other conditions being equal, leads to elongation of the initial sections for the velocities of both phases and the concentration of the admixture, as well as to an increase in the range of the jet.

The results of calculations presented in Fig. 2 make it possible to conclude that model (14) adequately describes the specific physical features of gas-dust flows and can be recommended for calculating flows with physicochemical processes.

NOTATION

U, V, longitudinal and transverse velocity components; x, r, space coordinates; R_0, radius of nozzle; ε, coefficient of turbulent viscosity; C_d, drag factor; Re_p, Reynolds number based on slip velocity ($\mathrm{Re}_p = (U - U_s)d/\nu$); $\mathrm{Sc_t}$, turbulent Schmidt number; d, diameter of particles; ρ_p, physical density of particles; ρ, density; ψ, stream function; ξ, ξ_1, ξ_2, transformed longitudinal coordinates; a_k, b_k, c_k, coefficients of series; k, z, constants; N, number density of particles. Subscripts: s refers to a solid phase; 0, parameter at the nozzle outlet; p, parameter determined from the physical characteristics of the admixture.

REFERENCES

1. G. N. Abramovich, Theory of Turbulent Jets [in Russian], Moscow (1984).
2. T. A. Girshovich, A. I. Kartushinskii, M. K. Laats, V. A. Leonov, and A. S. Mul'gi, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5, 26-31 (1981).
3. M. K. Laats and F. A. Frishman, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2, 186-191 (1970).
4. M. K. Laats and F. A. Frishman, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2, 153-157 (1973).
5. A. I. Kartushinskii, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1, 36-41 (1984).
6. A. A. Shraiber, L. B. Gavin, V. A. Naumov, et al., Turbulent Flows of Gas Suspension [in Russian], Kiev (1987).
7. R. I. Nigmatulin, Dynamics of Multiphase Media, Pt. 1 [in Russian], Moscow (1991).
8. A. A. Samarskii and Yu. P. Popov, Difference Schemes of Gas Dynamics [in Russian], Moscow (1975).